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MATHEMATICS

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1 CHAPTER

Relations and Functions

Q.1 Let $f : N \to N$ be defined by

$$f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$$ for all $n \in N$. State whether the function f is bijective. [CBSE 2009]

Q.2 Let $f : R \to R$ be defined as $f(x) = 10x + 7$. Find the function $g : R \to R$ such that $gof = fog = I_R$. [CBSE 2011]

Q.3 Find $fog(x)$ if $f(x) = |x|$ and $g(x) = |5x - 2|$ [CBSE 2011]

Q.4 Define a binary operation '$*$' on the set $\{0, 1, 2, 3, 4, 5\}$ as

$$a * b = \begin{cases} a+b & \text{if } a+b < 6 \\ a+b-6 & \text{if } a+b \geq 6 \end{cases}.$$

Show that zero is the identity for this operation and each element a of the given set is invertible with $6 - a$ being the inverse of a. [CBSE 2011]

Q.5 Consider the binary operations $* : R \times R \to R$ and $o : R \times R \to R$ defined as $a * b = |a - b|$ and $a\ o\ b = a$ for $a, b \in R$. Show that '$*$' is commutative but not associative, 'o' is associative but not commutative. Futher show that $\forall\ a, b, c \in R$ $a*(boc) = (a*b)o(a*c)$. [CBSE 2012]

Q.6 Let $f : W \to W$ be defined as $f(n) = n - 1$, if n is odd and $f(n) = n + 1$, if n is even. Show that f is invertible. Find the inverse of f. Here, W is the set of all whole numbers. [CBSE 2012, 2015]

Q.7 Consider $f : R_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with the inverse (f^{-1}) of f given by f^{-1} of f given by $f^{-1}(y) = \sqrt{y-4}$, where R_+ is the set of all non-negative real numbers. [CBSE 2013]

Q.8 Prove that the relation R in the set $A = \{1, 2, 3, ..., 12\}$ given by $R = \{(a, b) : |a - b|$ is divisible by $3\}$, is an equivalence relation. Find all elements related to the element 1. [CBSE 2013]

Q.9 If $R = \{(x, y) : x + 2y = 8)$ is a relation on N, write the range of R. [CBSE 2014]

Q.10 Let $R = \{(a, a^3) : a$ is a prime number less than $5\}$ be a relation. Find the range of R. [CBSE 2014]

Q.11 Let N denote the set of all natural numbers and R be the relation on N × N defined by (a, b) R (c,d) if $ad(b + c) = ad(b + c) = bc(a + d)$. Show that R is an equivalence relation. [CBSE 2015]

Q.12 Let $f : R - \left\{-\frac{4}{3}\right\} \to R$ be a function defined as $f(x) \frac{4x}{3x+4}$. Show that, in $f : R - \left\{-\frac{4}{3}\right\} \to$ Range of f, f is one-one and onto. Hence find f^{-1} : Range $f \to R - \left\{-\frac{4}{3}\right\}$ [CBSE 2017]

Q.13 Let $A = \{1,2,3,4\}$. Let R be the equivalence relation A × A defined by (a, b) R (c, d) iff $a + b = b + c$. Find the equivalence class [CBSE 2018]

Q.14 For each binary operation * defined below, determine whether * is commutative or associative. [CBSE 2019]

i) On Z, define $a * b = a - b$

ii) On Q, define $a * b = ab + 1$

iii) On Q, define $a * b = ab^2$

iv) On Z^+, define $a * b = 2ab$

v) On Z^+, define $a * b = ab$

vi) On $R - \{-1\}$, define $a * b = a/b + 1$

Q.15 Find which of the binary operations are commutative and associative. Does there exist identity for this binary operation on Q ? [CBSE 2019]

Q.16 Show that the relation R in the set Z of integers given by $R = \{(a, b) : 2$ divides $(a - b)\}$ is an equivalence relation. [CBSE 2019]

Q.17 Show that the relation R on the set Z of all integers, given by $R = \{(a,b) : 2$ divides $(a - b)$ is an equivalence relation. [CBSE 2019]

Q.18 Prove that the function $f : N \to N$, defined by $f(x) = x^2 + x + 1$ is one-one but not onto. Find inverse of $f : N \to S$, where S is range of f. [CBSE 2019]

2 CHAPTER

Inverse Trigonometric Functions

Q.1 Write the principal value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$.

(A) ..... (B) ..... (C) ..... (D) .....

[CBSE 2009]

Q.2 If $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$ is equal to [CBSE 2012]

(A) π (B) $\frac{-\pi}{3}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

Q.3 Prove the following :

$$\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right).$$

[CBSE 2012]

Q.4 Write the value of $\cot(\tan^{-1} a + \cot^{-1} a)$.

[CBSE 2012]

Q.5 Find the value of the $\tan$

$$\frac{1}{2}\left(\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right), |x|<1, y>0 \text{ and } xy<1.$$

[CBSE 2013]

Q.6 If $2\tan^{-1}(\cos\theta) = \tan^{-1}(2\operatorname{cosec}\theta)$, then show that [CBSE 2013]

Q.7 Write the principal value of $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$

[CBSE 2013]

Q.8 Write the value of [CBSE 2013]

Q.9 Show that : $\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$ [CBSE 2013]

Q.10 Prove that :

$$\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x - \frac{1}{\sqrt{2}} \le x \le 1$$

[CBSE 2014]

Q.11 Write the principal value of $\sin^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$

[CBSE 2014]

Q.12 Prove that : $\cot^{-1}7 + \cot^{-1}8 + \cot^{-1}18 = \cot^{-1}3$.

[CBSE 2014]

Q.13 Evaluate : $\tan\left\{2\tan^{-1}\left(\frac{1}{5}\right) + \frac{\pi}{4}\right\}$ [CBSE 2015]

Q.14 Prove that : $\cot^{-1}\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}, x \in \left(0, \frac{\pi}{4}\right)$

[CBSE 2016]

Q.15 Prove that : $\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$

[CBSE 2016, 2017]

Q.16 If $\sin^{-1}\left(\frac{3}{x}\right) + \sin^{-1}\left(\frac{4}{x}\right) = \frac{\pi}{2}$ then find value of x.

[CBSE 2019]

Q.17 If $\tan^{-1}x - \cot^{-1}x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$, $x > 0$ find the value of x and have find the value of $\sec^{-1}\left(\frac{2}{x}\right)$.

[CBSE 2019]

Q.18 Prove that : $\sin^{-1}\left(\frac{8}{17}\right) + \cos^{-1}\left(\frac{4}{5}\right) = \cot^{-1}\frac{36}{77}$

[CBSE 2019]

Q.19 Find the value of $\sin\left(\cos^{-1}\left(\frac{4}{5}\right) = \tan^{-1}\left(\frac{2}{3}\right)\right)$

[CBSE 2019]

Q.20 Prove the $\sin^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{5}{12}\right) + \cos^{-1}\left(\frac{63}{65}\right) = \frac{\pi}{2}$

[CBSE 2019]

Q.21 Prove that : $\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3} = \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$

[CBSE 2020]

Q.22 $\tan^{-1}3 + \tan^{-1}\lambda = \tan^{-1}\left(\frac{3+\lambda}{1-3\lambda}\right)$ is valid for what values of λ ? [CBSE 2020]

Q.23 $\tan^{-1}\left(\sin^{-1}\frac{3}{5} + \tan^{-1}\frac{3}{4}\right)$ is equal to [CBSE 2020]

Q.24 If $\cos\left(\sin^{-1}\frac{2}{\sqrt{5}} + \cos^{-1}x\right) = 0$, then x is equal to

[CBSE 2020]

Q.25 Find the value of $\sin^{-1}\left[\sin\left(-\frac{17\pi}{8}\right)\right]$

[CBSE 2020]

Q.26 Solve for x : $\sin^{-1}(1-x) - 2\sin^{-1}(x) \frac{\pi}{2}$ [CBSE 2020]

3 CHAPTER

Matrices

Q.1 If a matrix has 18 elements what Are the possible order it can have ? What if it has 5 elements. [CBSE 2011]

Q.2 Find the values of x, y and z if $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$ [CBSE 2011]

Q.3 Simplify :

$$\cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$$ [CBSE 2012]

Q.4 If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then find $A^T - B^T$. [CBSE 2012]

Q.5 If matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^2 = kA$, then write the value of k. [CBSE 2013]

Q.6 Find the values of a, b, c and d if

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$ [CBSE 2013]

Q.7 For what value of x, is the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ a skew-symmetric matrix ? [CBSE (Al)2013]

Q.8 If $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$, then find the matrix A. [CBSE 2013]

Q.9 If A is a 3×3 matrix, whose elements are given by $a_{ij} = \frac{1}{2}|3j + j|$, then write the value of a_{23}. [CBSE 2013]

Q.10 If $\begin{bmatrix} x-y & z \\ 2x-y & \omega \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$, find the value of $x + y$ [CBSE 2014]

Q.11 Write the element a_{23} of a 3×3 matrix $A = (a_{ij})$ whose elements a_{ij} are given by $a_{ij} = \frac{|i-j|}{2}$ [CBSE 2015]

Q.12 Find the value of $x + y$ from the following equation :

$$\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$ [CBSE 2015, 2017]

Q.13 Matrix $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is given to be symmetric, find values of a and b. [CBSE 2016]

Q.14 If $A = \begin{pmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{pmatrix}$ and $BA = (b_{ij})$, find $b_{21} + b_{32}$ [CBSE 2016]

Q.15 Write the number of all possible matrices of order 2×2 with each entry 1, 2 or 3. [CBSE 2016]

Q.16 Let A and B are matrices of order 3×2 and 2×4 respectively. Write the order of matrix (AB). [CBSE 2017]

Q.17 Show that all the diagonal elements of a skew symmetric matrix are zero. [CBSE 2017]

Q.18 Show that the elements along the main diagonal of a skew symmetric matrix are all zero. [*CBSE 2017*]

Q.19 If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew symmetric, find the values of 'a' and 'b' [*CBSE 2018*]

Q.20 If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 0 & 0 \end{bmatrix}$ is skew symmetric, find the values of 'a' and 'b'. [*CBSE 2018*]

Q.21 If $A = \begin{bmatrix} 3 & 9 & 0 \\ 1 & 8 & -2 \\ 7 & 5 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 & 2 \\ 7 & 1 & 4 \\ 2 & 2 & 6 \end{bmatrix}$. Then find the matrix B'A'. [*CBSE 2019*]

Q.22 Find the value of $(x - y)$ from the matrix equation.

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} -3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$ [*CBSE 2019*]

Q.23 If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then find the value of k, a and b. [*CBSE 2019*]

Q.24 If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then find the value of k, a and b. [*CBSE 2019*]

Q.25 Find the inverse of the following matrix using elementary transformations :

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$$ [*CBSE 2019*]

Q.26 If A and B are symmetric matrices, such that AB and BA are both defined, then prove that AB –BA is a skew-symmetric matrix. [*CBSE 2019*]

Q.27 For the matrix $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$, find $A + A^T$ and verify it is a symmetric matrix. [*CBSE 2019*]

Q.28 Given a skew-symmetric matrix $A = \begin{bmatrix} 0 & a & 1 \\ 7 & 1 & 1 \\ -1 & c & 0 \end{bmatrix}$, then value of $(a + b + c)^2$ is ..... [*CBSE 2020*]

Q.29 If $A + B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $A - 2B = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$, then A = ... [*CBSE 2020*]

Q.30 Construct a 2×2 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = |(i)^2 - j|$ [*CBSE 2020*]

4 CHAPTER

Determinants

Q.1 Write the value of the following determinant

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}.$$ [CBSE 2009]

Q.2 Using properties of determinants, solve the following for x :

$$\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0.$$ [CBSE 2011]

Q.3 Write the value of the following determinant.

$$\begin{bmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{bmatrix}$$ [CBSE 2012]

Q.4 If A is a square matrix and $|A| = 2$, then write the value of $|AA'|$, where A' is the transpose of matrix A. [CBSE 2013]

Q.5 Using properties of determinants, prove that

$$\begin{vmatrix} 2y & x-y-z & 2y \\ 2z & 2z & x-y-z \\ x-y-z & 2x & 2x \end{vmatrix} = (x+y+z)^3$$ [CBSE 2014]

Q.6 Write the value of $\Delta = \begin{bmatrix} x-y & x-z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{bmatrix}$ [CBSE 2015]

Q.7 If A is a 3×3 invertible matrix, then what will be the value of k if $\det(A^{-1}) = (\det A)^k$. [CBSE 2017]

Q.8 Using properties of determinants, prove that

$$\begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3z & 1 \end{vmatrix} = 9(3xyz + xy + yz + zx)$$ [CBSE 2018]

Q.9 Using properties of determinants, prove that

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ a & c & a+b \end{vmatrix} = 4abc$$ [CBSE 2019]

Q.10 If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}$, find A^{-1} ? [CBSE 2019]

Q.11 Hence solve the system of equations :

$x + y + z = 6$

$x + 3z = 11$

and $x - 2y + z = 0$ [CBSE 2019]

Q.12 If A is a square matrix of order 3 with $|A| = 4$, then write the value of $|-2A|$. [CBSE 2019]

Q.13 Using properties of determinants, prove that

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

[CBSE 2019]

Q.14 If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, find A^2 and show that $A^2 = A^{-1}$.

[CBSE 2019]

Q.15 Using matrix method, solve the following system of equations :

$2x - 3y + 5z = 13$

$3x + 2y - 4z = -2$

$x + y - 2z = -2$

$x = 1, y = 1, z = 3$ [CBSE 2019]

Q.16 Using properties of determinants, find the value of x for which

$$\begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$$ [CBSE 2019]

Q.17 If a, b, c are pth, qth and rth terms respectively of a G.P., then prove that

$$\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$$ [CBSE 2020]

Q.18 If $\begin{bmatrix} 2x & -9 \\ -2 & x \end{bmatrix} = \begin{bmatrix} -4 & 8 \\ 1 & -2 \end{bmatrix}$, then value of x is .....

[CBSE 2020]

Q.19 If A and B are square matrices of order 3 and $|A| = 5$, $|B| = 3$, then the value of $|3AB|$ is .....

[CBSE 2020]

5 CHAPTER

Continuity and Differentiability

Q.1 Find the value of a and b such that the function $f(x)$ defined by

$$f(x)\begin{cases} 5; & \text{if} \quad x \le 2 \\ ax+b; & \text{if} \quad 2 < x < 10 \\ 21 & \text{if} \quad x \ge 10 \end{cases}$$ is a continuous function

[CBSE 2011]

Q.2 If $x = \sqrt{a^{\sin^{-1}t}}$ and $y = \sqrt{a^{\cos^{-1}t}}$, show that $\frac{dy}{dx} = -\frac{y}{x}$

[CBSE 2012]

Q.3 If $y = (\tan^{-1}x^2)$, show that $(x^2+1)^2 y_2 + 2x(x^2+1)y_1 = 2$.

[CBSE 2012]

Q.4 If $y = \cos^{-1}\frac{\sqrt{a+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}, 0 < x < \frac{\pi}{2}$ then find the value of $\frac{dy}{dx}$.

[CBSE 2013]

Q.5 If $y = x^{\sin x} + (\sin x)^{\cos x}$, find $\frac{dy}{dx}$.

[CBSE 2013]

Q.6 For what value of 'k' is the function

$$f(x)\begin{cases} \frac{\sin 5x}{3x} + \cos x, & \text{if} \quad x \ne 3 \\ k, & \text{if} \quad x = 0 \end{cases}$$ constinuous at $x = 0$?

[CBSE 2017]

Q.7 Determine value of the constant 'k' so that the function

$$f(x)\begin{cases} \frac{kx}{|x|}, & \text{if} \quad x < 0 \\ 3, & \text{if} \quad x \ge 0 \end{cases}$$ constinuous at $x = 0$.

[CBSE 2017]

Q.8 Find 'a' and 'b', if the function given by

$$f(x) = \begin{cases} ax^2 + b, & \text{if } x < 1 \\ 2x+1, & \text{if } x \ge 1 \end{cases}$$ is differentiable at $x = 1$

[CBSE 2018]

Q.9 Find $\sin^{-1}\left[\frac{2^{x+1}}{1+4^x}\right]$.

[CBSE 2019]

Q.10 Find $\frac{dy}{dx}$, if $xy^2 - x^2 = 4$.

[CBSE 2019]

Q.11 If $\sin y = x \sin(a+y)$, prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$.

[CBSE 2019]

Q.12 If $y = x^{\cos x} + (\sin x)^{\cos x}$, find $\frac{dy}{dx}$.

[CBSE 2019]

Q.13 If $y = 2\sqrt{\sec(e^{2x})}$; then find $\frac{dy}{dx}$.

[CBSE 2019]

Q.14 If $y = \text{cosec}\,(\cot\sqrt{x})$, then find $\frac{dy}{dx}$.

[CBSE 2019]

Q.15 If $y = \log(\cos e^x)$, then find $\frac{dy}{dx}$.

[CBSE 2019]

Q.16 The function $f : R \to R$ given by $f(x) = -|x-1|$ is

(A) constinuous as well as differentiable at $x = 1$

(B) not continuous but differentiable at $x = 1$

(C) continuous but not differentiable at $x = 1$

(D) neither continuous nor differentiable at $x = 1$

[CBSE 2020]

Q.17 The function $f(x) = \dfrac{x-1}{x(x^2-1)}$ is discontinuous at

(A) exactly on point

(B) exactly two point

(C) exactly three point

(D) no point [*CBSE 2020*]

Q.18 If the function f defined as $f(x)$

$$\begin{cases} \dfrac{x^2-9}{x-3}, & x \neq 3 \\ k, & x = 3 \end{cases}$$ is continuous at $x = 3$, find the value of k. [*CBSE 2020*]

Q.19 If $x = a\cos\theta$; $y = b\sin\theta$, then find $\dfrac{d^2y}{dx^2}$ [*CBSE 2020*]

Q.20 If $\tan^{-1}\left(\dfrac{y}{x}\right) = \log\sqrt{x^2+y^2}$, prove that $\dfrac{dy}{dx} = \dfrac{x+y}{x-y}$ [*CBSE 2020*]

Q.21 Verify Rolle's theorem for the function $f(x) = e^x \cos x$ in $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$ [*CBSE 2020*]

6 CHAPTER

Application of Derivatives

Q.1 Show that the semi-vertical angle of the right circular cone of given total surface area and maximum volume is $\sin^{-1}\frac{1}{3}$. [*CBSE 2008*]

Q.2 Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$. [*CBSE 2008, 2014*]

Q.3 The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When $x = 8$ cm and $y = 6$ cm, find the rate of change of (*a*) the perimeter (*b*) the area of the rectangle. [*CBSE 2009*]

Q.4 Seand is pouring from a pipe at the rate of 12 cm^3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm ? [*CBSE 2011*]

Q.5 Prove that $y = \frac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function of θ in $\left[0, \frac{\pi}{2}\right]$. [*CBSE 2011, 2016*]

Q.6 If the radius of a sphere is measured as 9 m with an error of 0.03 m, then find the approximate error in calculating its surface area. [*CBSE 2011*]

Q.7 Show that the right-circular cone of least curved surface area and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base. [*CBSE 2011*]

Q.8 A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall at the rate of 2 m/sec. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall ? [*CBSE 2012*]

Q.9 Find the point on the curve $y = x^3 - 11x + 5$ at which the equation of tangent is $y = x - 11$. [*CBSE 2012*]

Q.10 Find the values of x for which $y = [x(x-2)]^2$ is an increasing function. [*CBSE 2014*]

Q.11 Find the equations of the tangent and normal to the curve $x = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$. [*CBSE 2014*]

Q.12 Show that a right circular cylinder of the given volume open at the top has minimum total surface area, provided its height is equal to the radius of the base. [*CBSE 2014*]

Q.13 Find the equation of tangent to the curve $y = \frac{x-7}{(x-2)(x-3)}$ at the point, where it cutes the x-axis. [*CBSE 2015*]

Q.14 Prove that $y = \frac{4\sin\theta}{(2+\cos\theta)} - \theta$ is an increasing function of θ in $\left[0, \frac{\pi}{2}\right]$. [*CBSE 2016*]

Q.15 Prove that the volume of the largest cone that can be inscribed in a sphere of radius a is $\frac{8}{27}$ of the volume of the sphere. [*CBSE 2016*]

Q.16 A window is in the form of rectangle surmounted by a semi-circular opening. Total perimeter of the window is 10 m. What will be the dimensions of the whole opening to admit maximum light and air through the whole opening? [CBSE 2017]

Q.17 The volume of a cube is increasing at the rate of 9 $cm^{3/s}$. How fast its surface area increasing when the length of an edge is 10 cm ? [CBSE 2017]

Q.18 Find the approximate change in the value of $\frac{1}{x^2}$, when x changes from $x = 2$ to $x = 2.002$. [CBSE 2018]

Q.19 Find the intervals in which the function $f(x) = -3\log(1+x) + 4\log(2+x) - \frac{4}{2+x}$ is strictly increasing or strictly decreasing. [CBSE 2018]

Q.20 Find whether the function $f(x) = \cos\left(2x + \frac{\pi}{4}\right)$; is increasing or decreasing in the interval $\left(\frac{3\pi}{8}, \frac{7\pi}{8}\right)$. [CBSE 2019]

Q.21 Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum voluem. [CBSE 2019]

Q.22 Find the minimum value of $(ax + by)$, where $xy = c^2$. [CBSE 2020]

Q.23 Find the dimensions of the rectangle of perimeter 36 cm which will sweep out a volume as large as possible, when revolved about one of its side. Also, find the maximum volume. [CBSE 2020]

7 CHAPTER — Integrals

Q.1 Integrate the following function : $\sqrt{4-x^2}$ [CBSE 2008]

Q.2 Integrate the following function : $\sqrt{1-4x^2}$ [CBSE 2008]

Q.3 Evaluate : $\int_0^{\pi} \frac{x\tan x}{\sec x + \tan x}\,dx$. [CBSE 2008]

Q.4 Evaluate : $\int x\sin^{-1}x\,dx$ [CBSE 2009]

Q.5 Evaluate : $\int \frac{e^{\tan^{-1}x}}{1+x^2}dx$ [CBSE 2011]

Q.6 Evaluate $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}}dx$. [CBSE 2011]

Q.6 Evalute : $\int \frac{x\cos^{-1}x}{\sqrt{1-x^2}}dx$ [CBSE 2014]

Q.7 Evalute : $\int_0^{\pi} \frac{x\tan x}{\sec x + \tan x}dx$ [CBSE 2014]

Q.9 Evaluate : $\int \frac{\sin(x-a)}{\sin(x+a)}\,dx$ [CBSE 2015]

Q.10 Evaluate : $\int \frac{x^2}{(x^2+4)(x^2+9)}\,dx$ [CBSE 2015]

Q.11 Evalute : $\int_{-1}^{3/2} |x\sin\pi x|\,dx$ [CBSE 2016]

Q.12 Find : $\int \frac{(2x-5)e^{2x}}{(2x-3)^3}\,dx$ [CBSE 2016]

Q.13 Find : $\int (2x+5)\sqrt{10-4x-3x^2}\,dx$ [CBSE 2016]

Q.14 Find $\int e^x \frac{\sqrt{1+\sin 2x}}{1+\cos 2x}\,dx$ [CBSE 2018]

Q.15 Find $\int e^x(\cos x - \sin x)\,\text{cosec}^2 x\,dx$ [CBSE 2019]

Q.16 Find $\int \frac{x-1}{(x-2)(x-3)}\,dx$ [CBSE 2019]

Q.17 Find $\int_{-\pi/4}^{0} \frac{1+\tan x}{1-\tan x}\,dx$ [CBSE 2019]

Q.18 Find $\int \frac{dx}{\sqrt{5-4x-2x^2}}$ [CBSE 2019]

Q.19 Evaluate $\int \frac{dx}{\sin(x-a)\cos(x-b)}$ [CBSE 2019]

Q.20 $\int x^2 e^{x^2}\,dx$ is equal to

(A) $\frac{1}{3}e^{x^3}+C$ (B) $\frac{1}{3}e^{x^4}+C$

(C) $\frac{1}{2}e^{x^3}+C$ (D) $\frac{1}{2}e^{x^2}+C$ [CBSE 2020]

Q.21 $\int_0^{\pi/8} \tan^2(2x)dx$ is equal to

(A) $\frac{4-\pi}{8}$ (B) $\frac{4+\pi}{8}$

(C) $\frac{4-\pi}{4}$ (D) $\frac{4-\pi}{2}$ [CBSE 2020]

Q.22 Evaluate : $\int \frac{dx}{9+4x^2}$ [CBSE 2020]

Q.23 Find : $\int \frac{2^{x+1}-5^{x-1}}{10^x}\,dx$ [CBSE 2020]

Q.24 If $\int_0^a \frac{dx}{1+4x^2} = \frac{\pi}{8}$, then find the value of a. [CBSE 2020]

Q.25 Find $\int \frac{\tan^3 x}{\cos^3 x}dx$ [CBSE 2020]

Q.26 Evaluate : $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx$ [CBSE 2020]

8 CHAPTER — Application of Integrals

Q.1 Find the area of the region enclosed between the two circles $x^2 + y^2 = 1$ and $(x - 1)^2 + y^2 = 1$. [CBSE 2008]

Q.2 Find the area of the region included between the parabola $4y = 3x^2$ and the line $3x - 2y + 12 = 0$. [CBSE 2009]

Q.3 Sketch the graph of $y = |x + 3|$ and evaluate the area under the curve $y = |x + 3|$ above x-axis and between $x = -6$ to $x = 0$. [CBSE 2011]

Q.4 Find the area of the region $\{(x, y) : x^2 + y^2 \le 4, x + y \ge 2\}$ [CBSE 2012]

Q.5 Find the area of the region included between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, where $a > 0$. [CBSE 2013]

Q.6 Find the area of the region included between the parabola $4y = 3x^2$ and the line $3x - 2y + 12 = 0$ [CBSE 2013]

Q.7 Find the area of the region bounded by the parabola $y^2 = 2x$ and the line $x - y = 4$. [CBSE 2013]

Q.8 Using the integration find the area of the triangle formed by positive x-axis and tangent and normal to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$. [CBSE 2015]

Q.9 Using integration, find the area bounded by the tangent to the curve $4y = x^2$ at the point (2, 1) and the lines whose equations are $x = 2y$ and $x = 3y - 3$. [CBSE 2016]

Q.10 Using integration find the area of the region $\{(x,y) : x^2 + y^2 \le 2ax, y^2 \ge ax, x, y \ge 0\}$ [CBSE 2016]

Q.11 Find the area of the region $\{(x, y) : x^2 + y^2 \le 1 \le x + y\}$. [CBSE 2017]

Q.12 Find the area of the region $\{(x, y) : y^2 \le 4x, 4x^2 + 4y^2 \le 9\}$. [CBSE 2017]

Q.13 Using integration, find the area of the following region: $\{(x, y) : |x - 1| \le y \sqrt{5 - x^2}\}$ [CBSE 2017]

Q.14 Using integration, find the area of the triangle ABC, co-ordinates of whose vertices are A(4,1), B(6,6) and C(8, 4). [CBSE 2017]

Q.15 Using the method of integration, find the area of the region bounded by the lines $3x - 2y + 1 = 0$, $2x + 3y - 21 = 0$ and $x - 5y + 9 = 0$ [CBSE 2019]

Q.16 Find the area of the triangle whose vertices are (–1,1),(0,5) and (3,2), using integration. [CBSE 2019]

Q.17 Using integration, find the area of the triangle whose vertices are (2,3), (3,5) and (4, 4) [CBSE 2019]

Q.18 Find the area of the following region using integration $\{(x, y) : y \le |x| + 2, y \ge x^2\}$. [CBSE 2020]

Q.19 Find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$ [CBSE 2020]

Q.20 Using integration find the area of the region bounded between the two circles $x^2 + y^2 = 9$ and $(x - 3)^2 + y^2 = 9$. [CBSE 2020]

9 CHAPTER — Differential Equations

Q.1 Solve $x dy - y dx = \sqrt{x^2 + y^2}\, dx$. [CBSE 2011]

Q.2 Form the differential equation of the family of parabolas having vertex at the origin and axis along positive y-axis. [CBSE 2011]

Q.3 Find the particular solution of the differential equation $x(x^2 - 1)\frac{dy}{dx} = 1$; $y = 0$ when $x = 2$. [CBSE 2012]

Q.4 Find the particular solution of the differential equation: $\frac{dy}{dx} + y \cot x = 4x \operatorname{cosec} x (x \ne 0)$, given that $y = 0$ when $x = \frac{\pi}{2}$ [CBSE 2012]

Q.5 From the differential equation of the family of circles in the second quadrant and touching the co-ordinate axes. [CBSE 2012]

Q.6 Solve the following differential equation :
$3e^x \tan dx + (2 - e^x) \sec^2 y\, dy = 0$, given that when $x = 0$, $y = \frac{\pi}{4}$ [CBSE 2012]

Q.7 Solve the differential equation :
$\tan^{-1} y - x)dy = (1 + y^2)dx$ [CBSE 2013]

Q.8 Show that the differential equation
$x\frac{dy}{dx}\sin\left(\frac{y}{x}\right) + x - y\sin\left(\frac{y}{x}\right) = 0$ is homogeneous. Find the particular solution of this differential equation, given that $x = 1$ when $x = 1$ when $y = \frac{\pi}{2}$. [CBSE 2013]

Q.9 Solve the following differential equation :
$\frac{dy}{dx} + 2y\tan x = \sin x$, given that $y = 0$, when $x = \frac{\pi}{3}$
[CBSE 2014]

Q.10 Find the particular solution of the differential equation
$\log\left(\frac{dy}{dx}\right) = 3x + 4y$, given that $y = 0$ when $x = 0$.
[CBSE 2014]

Q.11 Solve the differential equation
$(1 + x^2)\frac{dy}{dx} + y = e^{\tan^{-1} x}$ [CBSE 2014]

Q.12 Solve the following differential equation :
$\frac{dy}{dx} + 2y\tan x = \sin x$, given that $y = 0$, when $x = \frac{\pi}{2}$
[CBSE 2014]

Q.13 Find the differential equation representing the curve $y = cx + c^2$. [CBSE 2015]

Q.14 Find the differential equation representing the curve $y = cx + c^2$. [CBSE 2015]

Q.15 Find the differential equation representing the curve $y = e^{-x} + ax + b$, where a and b are arbitrary constants. [CBSE 2015]

Q.16 Find the differential equation representing the family of curves $\upsilon = \frac{A}{r} + B$, where A and B are arbitrary constants. [CBSE 2015]

Q.17 Find the differential equation for all the straight lines, which are at a unit distance from the origin. [CBSE 2015]

Q.18 Show that the differential equation $2ye^{x/y}\,dx + (y - 2xe^{x/y})\,dy = 0$ is homogeneous and find its particular solution, given that $x = 0$ when $y = 0$ [CBSE 2016]

Q.19 Solve : $x\frac{dy}{dx} + y - x + xy\cot x = 0 (x \neq 0)$ [CBSE 2016]

Q.20 Solve the following differential equation :
$(\cot^{-1} y + x)\,dy = (1 + y^2)\,dx$ [CBSE 2016]

Q.21 Show that the differential equation
$(x - y)\frac{dy}{dx} = x + 2y$ is homogeneous and solve it
[CBSE 2017]

Q.22 Degree of differential equation
$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 + y = 0$ is [CBSE 2019]

Q.23 For a differential equation representing the faimily of curves $y = A\sin x$, by eliminating the arbitrary constant.
[CBSE 2019]

Q.24 Find the differential equation of the family of curves represented by $y^2 = a(b^2 - x^2)$. [CBSE 2019]

Q.25 Solve the differential equation $(y + 3x^2)\frac{dx}{dy} = x$.
[CBSE 2019]

Q.26 Find the integrating factor of the differential equation
$x\frac{dy}{dx} - 2y = 2x^2$. [CBSE 2019]

Q.27 Solve the differential equation :
$(x - y)\,dy - (x + y)\,dx = 0$. [CBSE 2019]

Q.28 Write the order and degree of the differential equation
$\left(\frac{d^4y}{dx^4}\right) = \left[x + \left(\frac{dy}{dx}\right)^2\right]^3$. [CBSE 2019]

Q.29 Form the differential equation representing the family of curves $y^2 = m(a^2 - x^2)$ by eliminating the arbitrary constant 'm' and 'a'. [CBSE 2019]

Q.30 Solve the differential equation
$\frac{dy}{dx} = \frac{1}{1 + x^2}$; $y(0) = 3$. [CBSE 2019]

Q.31 The degree of the differential equation
$x^2\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^3$ is [CBSE 2020]
(A) 1 (B) 2 (C) 3 (D) 6

Q.32 Solve the following differential equation :
$(1 + e^{y/x})\,dy + e^{y/x}\left(1 - \frac{y}{x}\right)dx = 0, (x \neq 0)$. [CBSE 2020]

Q.33 Find the general solution of the differential equation $x^2y\,dx - (x^3 + y^3)\,dy = 0$. [CBSE 2020]

10 CHAPTER — Vector Algebra

Q.1 Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2 respectively having $\vec{a}\cdot\vec{b} = \sqrt{6}$. [CBSE 2011]

Q.2 Write the projection of the vector $\hat{i}-\hat{j}$ on the vector $\hat{i}+\hat{j}$. [CBSE 2011]

Q.3 Using vectors, find t he area of the triangle with vertices A(1, 1, 2), B(2, 3, 5) and C (1, 5, 5). [CBSE 2011]

Q.4 Write the position vector of the mid-point of the vector joining the points P(2, 3,4) and Q(4,1, –2). [CBSE 2011]

Q.5 Find the scalar components of the vector $\overrightarrow{AB}$ with initial point A(2,1) and terminal point B(–5, 7). [CBSE 2012]

Q.6 The two adjacent sides of a parallelogram are $2\hat{i}-4\hat{j}+5\hat{k}$ and $\hat{i}-2\hat{j}-3\hat{k}$. Find the unit vector parallel to one of its diagonals. Also, find its area. [CBSE 2012]

Q.7 Write the value of the area of the parallelogram determined by the vectors $2\hat{i}$ and $3\hat{j}$. [CBSE 2012]

Q.8 The magnitude of the vector product of th vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to $\sqrt{2}$. Find the value of λ. [CBSE 2013]

Q.9 The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with the unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ. [CBSE 2014]

Q.10 Find a unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$, where $\vec{a}=\hat{i}-7\hat{j}+7\hat{k}$ and $\vec{b}=3\hat{i}-2\hat{j}+2\hat{k}$. [CBSE 2015]

Or

Show that the vectors $\vec{b}=3\hat{i}-2\hat{j}+2\hat{k}$, $-2\hat{i}+3\hat{j}-4\hat{k}$ and $\hat{i}-3\hat{j}+5\hat{k}$ are coplanar.

Q.11 If $|\vec{a}| = a$, then find the value of the following :

$|\vec{a}\times\hat{i}|^2+|\vec{a}\times\hat{j}|^2+|\vec{a}\times\hat{k}|^2$ [CBSE 2015]

Q.12 Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be there vectors such that $|\vec{a}|=1$, $|\vec{b}|=2$ and $|\vec{c}|=3$. If the projection of $\vec{b}$ along $\vec{a}$ is equal to projection of $\vec{c}$ along $\vec{a}$; and $\vec{b}$, $\vec{c}$ are perpendicular to each other, then find $|3\vec{a}-2\vec{b}+2\vec{c}|$. [CBSE 2015]

Q.13 Find a unit vector perpendicular to the plane of triangle ABC, where the coordinates of its vectices are A(3, –1,2), B(1, –1, –3) and C(4, –3,1) [CBSE 2015]

Q.14 Write the number of vectors of unit length perpendicular to both the vectors $\vec{a}=2\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=\hat{j}+\hat{k}$ [CBSE 2016]

Q.15 Show that the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ are coplanar, iff $\vec{a}+\vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are coplanar. [CBSE 2016]

Q.16 If $\vec{a}$ and $\vec{b}$ are unit vectors, then what is the angle between $\vec{a}$ and $\vec{b}$ for $\vec{a}-\sqrt{2}\,\vec{b}$ to be a unit vector? [CBSE 2016]

Q.17 The *x*-coordinate of a point on the line joining the point P(2,2,1) and Q(5,1, –2) is 4. Find its z-coordinate. [CBSE 2017]

Q.18 If $\vec{a}$, $\vec{b}$, $\vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$. Also, find the angle which $\vec{a}+\vec{b}+\vec{c}$ makes with $\vec{a}$ or $\vec{b}$ or $\vec{c}$ [CBSE 2017]

Q.19 Find the area of a parallelogram ABCD whose side AB and the diagonal AC are given by the vectors $3\hat{i}+\hat{j}+4\hat{k}$ and $4\hat{i}+5\hat{k}$ respectively. [CBSE 2017]

Q.20 Using vectors find the area of triangle ABC with vertices A(1,2,3), B(2, –1,4) and C(4, 5, –1). [CBSE 2017]

Q.21 If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then prove that $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$ and hence show that $[\vec{a}\ \ \vec{b}\ \ \vec{c}\,]=0$ [CBSE 2018]

Q.22 Using vectors, prove that the points (2, –1, 3), (3,–5,1) and (–1,11,9) are collinear. [CBSE 2019]

Q.23 X and Y are two points with position vectors $3\vec{a}+\vec{b}$ and $\vec{a}-3\vec{b}$ respectively. Write the position vector of a point Z which divides the line segment XY in the ratio 2 : 1 externally. [CBSE 2019]

Q.24 Let $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$ be two vectors. Show that the vector $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ are perpendicular to each other. [CBSE 2019]

Q.25 For any two vectors $\vec{a}$ and $\vec{b}$, prove that $(\vec{a}\times\vec{b})^2=\vec{a}^2-\vec{b}^2-(\vec{a}\cdot\vec{b})^2$ [CBSE 2019]

Q.26 The vectors $3\hat{i}-\hat{j}+2\hat{k}$, $2\hat{i}+\hat{j}+3\hat{k}$ and $\hat{i}+\lambda\hat{j}-\hat{k}$ are coplanar if.
(A) –2
(B) 0
(C) 2
(D) Any real number [CBSE 2020]

Q.27 Show that for any two non-zero vectors $\vec{a}$ and $\vec{b}$, $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$ iff $\vec{a}$ and $\vec{b}$ are perpendicular vectors. [CBSE 2020]

Q.28 If $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$ represent two adjacent sides of a parallelogram, find unit vectors parallel to the diagonals of the parallelogram. [CBSE 2020]

Q.29 Using vectors, find the area of the triangle ABC with vertices A(1,2,3), B(2,–1,4) and C(4,5, –1) [CBSE 2020]

11 CHAPTER Three Dimensional Geometry

Q.1 The vector equation of two lines are

$\vec{r}=(1-t)\hat{i}+(t-2)\hat{j}+(3-2t)\hat{k}$ and

$\vec{r}=(s+1)\hat{i}+(2s-1)\hat{j}-(2s+1)\hat{k}$.

Find the shortest distance between these lines. [CBSE 2011]

Q.2 Write the direction consine of a line equally inclined to the three coordinate axes. [CBSE 2011]

Q.3 Find the coordinates of the point where the line through the points (3, – 4, – 5) and (2, – 3, 1) crosses the plane $2x + y + z = 7$. [CBSE 2012]

Q.4 If a line has direction ratios 2, –1, –2, then what are its direction consines ? [CBSE 2012]

Q.5 Write the direction cosines of a line parallel to z-axis. [CBSE 2012]

Q.6 Find the equation of the plane which contains the line of intersection of the planes $\vec{r}.(\hat{i}+2\hat{j}+3\hat{k})-4=0$ and $\vec{r}.(2\hat{i}+\hat{j}-\hat{k})+5=0$ and which is perpendicular to the plane $\vec{r}.(5\hat{i}+3\hat{j}-6\hat{k})+8=0$. [CBSE 2013]

Q.7 Write the cartesian equation of a plane, bisecting the line segment joining the points A(2,3,5) and B(4,5,7) at right angles. [CBSE 2013]

Q.8 Find the coordinates of the point, where the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}$ intersects the plane $x - y + z - 5=0$. Also find the angle between the line and the plane. [CBSE 2013]

Q.9 Find the distance of the point (–1, –5, –10) from the point of intersection of the line $\vec{r}=(2\hat{i}-\hat{j}+2\hat{k})+\lambda(3\hat{i}+4\hat{j}+2\hat{k})$ and the plane $\vec{r}.(\hat{i}-\hat{j}+\hat{k})=5$ [CBSE 2014]

Q.10 Write the vector equation of the plane, passing through the point (a,b,c) and parallel to the plane $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=2$. [CBSE 2014]

Q.11 Find the shortest distance between the following lines:

$\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$ and $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$ [CBSE 2014]

Q.12 Show that the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}$ intersect. Also find their point of intersection. [CBSE 2014]

Q.13 Find the distance between the point (7,2,4) and the plane determine by the points A(2,5,–3), B(–2,–3,5) and C(5,3,–3) [*CBSE 2014*]

Q.14 Find the sum of the intercepts cut off by the plane $2x + y - z$, on the coordinate axes. [*CBSE 2015*]

Q.15 A line passing through the point A with position vector $\vec{a} = 4\hat{i} + 2\hat{j} + 2\hat{k}$ is parallel to the vector $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$. Find the length of the perpendicular drawn on this line from a point P with position vector $\vec{r}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$ [*CBSE 2015*]

Q.16 From the point P(a,b,c), perpendiculars PL and PM are drawn to YZ and ZX planes respectively. Find the equation of the plane OLM. [*CBSE 2015*]

Q.17 Find the distance of the point (1, –2,3) from the plane $x - y + z = 5$ measured parallel to the line whose direction cosines are proportional to 2, 3, – 6. [*CBSE 2015*]

Q.18 Write the coordinates of the point which is the reflection of the point (α,β,γ) in the XZ-plane. [*CBSE 2016*]

Q.19 Write the equation of a plane which is at a distance of $5\sqrt{3}$ units from origin and the normal to which is equally inclined to coordinate axes. [*CBSE 2016*]

Q.20 Find the coordinates of the point where the line through the points A(3,4,1) and B(5,1,6) crosses the XZ plane. Also find the angle which this line makes with the XZ plane. [*CBSE 2016*]

Q.21 Find the coordinate of the point P where the line through A(3, –4, –5) and B(2, –3,1) crosses the plane passing through three points L(2,2,1), M(3,0,1) and N(4,–1,0). Also, find the ratio in which P divides the line segment AB. [*CBSE 2016*]

Q.22 Prove that the line through A(0,–1,–1) and B(4, 5,1) intersects the line through C(3,9,4) and D(–4,4,4). [*CBSE 2016*]

Q.23 Find the coordinates of the foot of perpendicular and perpendicular distance from the point P(4,3,2) to the plane $x + 2y + 3z = 2$. Also find the image of P in the plance. [*CBSE 2016*]

Q.24 If a line makes angles 90° and 60° respectively with the positive directions of x and y axes, find the angle which its makes with the positive direction of z-axis. [*CBSE 2017*]

Q.25 Find the vector equation of the line passing through the point A(1, 2, –1) and parallel to the line $5x - 25 = 14 - 7y = 35z$ [*CBSE 2017*]

Q.26 Find the vector and cartesian equations of the line passing through the point (1, 2, –4) and perpendicular to the two line $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{3} = \frac{z-5}{-5}$. [*CBSE 2017*]

Q.27 A variable plane which remains at a constant distance $3p$ from the origin cuts the coordinate axes at A, B, C. Show that the locus of the centroid of triangle ABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$. [*CBSE 2017*]

Q.28 Find the equation of the line which intersects the lines $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and passes through the point (1,1,1). [*CBSE 2018*]

Q.29 Find the co-ordinate of the point, where the line $\frac{x+2}{1} = \frac{y-5}{3} = \frac{z+1}{5}$ cuts the yz-plane. [*CBSE 2019*]

Q.30 Find the angle between the line $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(3\hat{i} - \hat{j} + 2\hat{k})$ and the plane $\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = 3$. [*CBSE 2019*]

Q.31 Find the equation of planes passing through the intersection of the planes $\vec{r}.(2\hat{i}+6\hat{j})+12=0$ and $\vec{r}.(3\hat{i}-\hat{j})+4\hat{k})=0$ and are at a unit distance from the origin. [*CBSE 2019*]

Q.32 Find the vector equation of the plane determined by the points A(3, –1,2) B(5,2,4) and C(–1, –1,6) . Hence, find the distance of the plane, thus obtained from the origin. [*CBSE 2019*]

Q.33 The co-ordinates of the foot of the perpendicular drawn form the point (2, –3, 4) on the *y*-axis is

(A) (2, 3, 4)

(B) (–2, –3, –4)

(C) (0,–3, 0)

(D) (2, 0, 4) [*CBSE 2020*]

Q.34 The two planes $x - 2y + 4z = 10$ and $18x + 17y + kz = 50$ are perpendicular, if k is equal to [*CBSE 2020*]

(A) –4 (B) 4

(C) 2 (D) –2

Q.35 Find the points of intersection of the line

$\hat{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda\ (3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane

$\hat{r}.(\hat{i} - \hat{j} + \hat{k}) = 5$ [*CBSE 2020*]

Q.36 Find the co-ordinates of the point where the line through (–1, 1, –8) and (5, –2, 10) crosses the ZX-plane. [*CBSE 2020*]

Q.37 Find the vector and cartesian equations of the line which is perpendicular to the lines with equations

$\frac{x+2}{1}=\frac{z-3}{2}=\frac{z+1}{4}$ and $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and passes through the point (1,1,1). Also find the angle between the given lines. [*CBSE 2020*]

Q.38 Show that the lines $\frac{x-2}{1}=\frac{y-2}{3}=\frac{z-3}{1}$ and $\frac{x-2}{1}=\frac{y-3}{4}=\frac{z-4}{2}$ intersect. Also, find the co-ordinate of the point of intersection and equation of the plane containing the two lines. [*CBSE 2020*]

12 CHAPTER

Linear Programming

Q.1 An aeroplane can carry a maximum of 200 passengers. A profit of Rs. 400 is made on each first class ticket and a profit of Rs. 300 is made on each second class ticket. The airline reserves atleast 20 seats for first class. However, atleast four times as many passengers prefer to travel by second class than by first class. Determine how many tickets of each type must be sold to maximize profit for the airline. Form an L.P.P. and solve it graphically. *[CBSE 2008]*

Q.2 A merchant planes to sell two types of personal computers-a desktop model and a portable model that will cost Rs. 25000 and Rs 40000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he doesn't want to invest more than Rs. 70 lakhs and if his profit on the desktop model is Rs. 4500 and on portable model is Rs. 5000. Make an L.P.P. and solve it graphically.

[CBSE 2011]

Q.3 A dietician wishes to mix two types of foods in such a way that the vutamin contents of teh mixture contains at least 8 units of A and 10 units of vitamin C. Food I contains 2 units / kg of vitamin A and 1 unit/kg of vitamin C while Food II contains 1 units / kg of vitamin A and 2 units/ kg of vitamin C. It costs Rs. 5 per kg to purchase Food I and Rs. 7 per kg to purchase Food II. Determine the minimum cost of such a mixture. Formulate the above as a LPP and solve it graphically.

[CBSE (Al) 2012]

Q.4 A manufacture produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to producea package of bolts. He earns a profit of Rs 17.50 per package on nuts and Rs. 7 per package on bolts. How many packages of each should be produced each day so as to maximize his profits if he operates his machines for at the most 12 hours a day ? Form the linear programming problem and solve it graphically.

[CBSE 2012]

Q.5 A dealer wishes to purchase a number of fans and sewing machines. He has only Rs.5760 to invest and has space for at the most 20 items. A fan consts him Rs.360 and a sweing machine Rs. 240. He expects to sell a fan at a profit of Rs. 22 and a sewing machine for a profit of Rs.18. Assuming that he can sell all the items that he buys, how should be invest his money to maximise his profit ? Solve it graphically.

[CBSE 2014]

Q.6 A dealer deals in two items only-item A and item B. He has Rs. 50,000 to invest and a space to store at most 60 items. An item A costs Rs. 2500 and an item B costs Rs.500. A net profit to him on item A is Rs. 500 and on item B Rs. 150. If he can sell all the items that he purchases, how should he invest his amount to have maximum profit. Formulate an LPP and solve it graphically. *[CBSE 2015]*

Q.7 Maximise $Z = 8x + 9y$ subject to the constraints given below :

$2x + 3y \le 6$; $3x - 2y \le 6$; $y \le 1$; $x, y \ge 0$. *[CBSE 2015]*

Q.8 Minimize and maximize $Z = 5x + 2y$ subject to the following constraints :

$x - 2y \le 2$, $3x + 2y \le 12$, $-3x + 2y \le 3$, $x \ge 0$, $y \ge 0$

[CBSE 2015]

Q.9 A diet is to contain at least 80 units of Vitamin A and 100 units of minerals. Two foods, F_1 and F_2 are available costing Rs. 5 per unit and Rs. 6 per unit respectively. One unit of food F_1 contains 4 units of vitamin A and 3 units of minerals whereas one unit of food F_2 contains 3 units of vitamin A and 6 units of minerals. Forulate this as a linear programming problem. Find the minimum cost of diet that consists of mixture of these two foods and also meets minimum nutritional requirement. [*CBSE 2016*]

Q.10 A retired person wants to invest an amount of Rs. 50,000. His broker recommends investing in two type of bonds 'A' and 'B' yielding 10 % and 9% return respectively on the invested amount. He decides to invest at least 20,000 in bond 'A' and at least 10,000 in bond 'B'. He also wants to invest at least as much in bond 'A' an in bond 'B'. Solve this linear programming problem graphically to maximise his returns.

[*CBSE 2016*]

Q.11 A manufacturer produces two products A and B. Both the products are processed on two different machines. The available capacity of first machine is 12 hours and that of second machine is 9 hours per day. Each unit of product A requires 2 hours on both machines and each unit of prodcut B requires 3 hours on both machines and each unit of product B requires 2 hours on first machine and 1 hour on second machine. Each unit of product A is sold at Rs. 7 profit and that of Rs.4. Find the production level per day for maximum profit graphically. [*CBSE 2016*]

Q.12 Two tailors, A and B, earn Rs. 300 and Rs. 400 per day respectively. A can stitch 6 shirts and 4 pairs of trousers while B can stitch 10 shirts and 4 pairs of trousers per day. To find how many days should each of them work and if it is desired to produce at least 60 shirts and 32 pairs of trousers at a minimum labour cost, formulate this as an LPP. [*CBSE 2017*]

Q.13 A company produces two types of goods A and B, that require gold and silver. Each unit of type A requires 3g of silver and 1 g of gold while that of type B requires 1 g of silver and 2 g of gold. The company can produce a maximum of 9 g of silver and 8 g of gold. If each unit of type A brings a profit of Rs. 40 and that of type B Rs.50, formulate LPP to maximize profit. [*CBSE 2017*]

Q.14 A company manufactures two types of novelty souvenirs made of plywood. Souvenirs to type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours for assembling. The prifit for type A souvenirs is Rs.100 each and for type B souvenirs profit is Rs.120 each. How many souvenirs of each type should the company manufacture in order to maximize the profit ? Formulate the problem as LPP and then solve it graphically.

[*CBSE 2019,2020*]

Q.15 A manufacturer has employed 5 skilled men and 10 semi-skilled men and makes two models A and B of an article. The making of one item of model A requires 2 hours work by a skilled man and 2 hours work by a semi-skilled man. One item of model B requires 1 hour by a skilled man and 3 hours by a semi-skilled man. No man is expected to work more then 8 hours per day. The manufacturer's profit on an item of model A is 15 and on an item of model B is 10. How many of items of each model should be made per day in order to maximize daily profit ? Formulate the above LPP and solve it graphically and find the maximum profit. [*CBSE 2019*]

Q.16 In an LPP, if the objective function $Z = ax + by$ has the same maximum value on two corner points of the feasible region, then the number of points of which Z_{max} occurs is [*CBSE 2020*]

(A) 0 (B) 2 (C) finite (D) infinite

Q.17 Solve the following LPP graphically :

Minimise $Z = 5x + 7y$

Subject to the constrainsts

$3x + y \geq 8$

$x + 2y \geq 10$

$x, y \geq 0$ [*CBSE 2020*]

13 CHAPTER

Probability

Q.1 Find the probability of throwing atmost 2 sixes in 6 throws of single die. [*CBSE 2011*]

Q.2 Bag I contains 3 red and 4 black balls and bag II contains 4 red and 5 block balls. One ball is transferred from bag I to bag II and then a ball is drawn from bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black. [*CBSE 2011*]

Q.3 Bag I contains 3 red and 4 black balls while another bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from bag II. [*CBSE 2011*]

Q.4 Suppose 5% of men and 0.25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male ? Assume that there are equal number of males and females. [*CBSE 2011*]

Q.5 Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin 3 times and notes the number of heads. If she gets 1, 2, 3 or 4 she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly on head, what is the probability that she threw 1, 2, 3 or 4 with the die ? [*CBSE 2012*]

Q.6 In a hockey match, both teams A and B scored same number of goals up to the end of the game, so to decide the winner, the referee asked both the captains to throw a die alternately and decided that the team, whose captian gets a six first, will be declared the winner. If the captain of team A was asked to start, find their respective probilities of winning the mtach and state whether the decision of the referee was fair or not. [*CBSE 2013*]

Q.7 Assume that the chances of a patient having a heart attack is 40%. Assuming that a meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drug reduces its chance by 25%. At a time a patient can choose any one of the two options with equal probabulities. It is given that after going through one of the two options, the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga. [*CBSE 2013,2015*]

Q.8 Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find E(X) or the mean of the distribution. [*CBSE (Al)2014*]

Q.9 Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that

i) the youngest is a girl ?

ii) atleast one is a girl ? [*CBSE 2014*]

Q.10 Two numbers are selected at random (without replacement) from the first six positive integers. Let X denotes the larger of two numbers obtained. Find E(X). Where E(X) is expectation of X. [*CBSE 2015*]

Q.11 A bag A contains 4 block and 6 red balls and bag B contains 7 black and 4 red balls. A die is thrown. If 1 or 2 appears on it, then bag A is chosen, otherwise bag B. If two balls are drawn at random (without replacement) from the selected bag, find the probability of one of the them being red and another black. [*CBSE 2015*]

Q.12 A committee of 4 students is selected at random from a group consisting of 7 boys and 4 girls. Find the probability that there are exactly 2 boys in the committee, then that at least one girl must be there in the committee. [*CBSE 2016*]

Q.13 Three persons A, B and C apply for a job of manager in a private company. Chances of their selection (A, B and C) are in the ratio 1 : 2 : 4. The probabilities that A, B and C can introduce changes to improve profits of the company are 0.8, 0.5 and 0.3 respectively. If the change does not take place, find the probability that it is due to the appointment of C. [*CBSE 2016*]

Q.14 A bag contains 4 balls. Two balls are drawn at random (without replacement) and are found to be white. What is the probability that all balls in the bag are white ? [*CBSE 2016*]

Q.15 Three number are selected at random (without replacement) form first six positive integers. If X denotes the smallest of the three numbers obtained, find the probability distribution of X. Also, find the mean and variance of the distribution. [*CBSE 2016*]

Q.16 An urn contains 3 white and 6 red balls. Four balls are drawn one by one with replacement from the run. Find the probability distribution of the number of red drawn. Also find mean and variance of the distribution. [*CBSE 2016*]

Q.17 A die, whose faces are marked 1,2,3 in red and 4,5,6 in green, is tossed. Let A be the event "number obtained is even" and B be the event "number obtained is red". Find if A and B are independent events. [*CBSE 2017*]

Q.18 Prove that if E and F are independent events, then the events E and F′ are are also independent. [*CBSE 2017*]

Q.19 Often it is taken that a truthful person commands, more respect in the society. A man is known to speak the truth 4 out of 5 times. He throws a die and reports that it is actually a six. Find the probability that it is actually a six. [*CBSE 2017*]

Q.20 There are 4 cards numbered 1 to 4, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two drawn cards. Find the mean and variance of X. [*CBSE 2017*]

Q.21 There are 4 cards numbered 1,3,5 and 7, one number on one card. Two cards are drawn at random without replacement. Let X denote he sum of the numbers on the two drawn cards. Find the mean and variance of X. [*CBSE 2017*]

Q.22 Four bad oranges are accidentally mixed with 16 good ones. Find the probability distribution of the number of bad oranges when two oranges are drawn at random from this lot. Find the mean and variance of the distribution. [*CBSE 2018*]

Q.23 If P(A) = 0.6, P(B) = 0.5 and P(B/A) =0.4, find P(A∪B) and P(A/B). [*CBSE 2019*]

Q.24 The probability of two students A and B coming to school in time are $\frac{2}{7}$ and $\frac{4}{7}$, respectively. Assuming that the events 'A coming on time' and 'B coming on time' are independent, find the probability of only one of them coming to school on time. [*CBSE 2019*]

Q.25 Four cards are drawn one by one with replacement from a well-shuffled deck of playing cards. Find the probability that at least three cards are of diamonds. [*CBSE 2019*]

Q.26 There are two boxes I and II. Box I contains 3 red and 6 black balls. Bos II contains 5 red and '*n*' block balls. One of the two boxes, box I and box II is selected at random and a ball is drawn at random. The ball drawn is found to be red. If the probability that this red ball comes out from box II is $\frac{3}{5}$, find the value if '*n*'. [*CBSE 2019*]

Q.27 A beg contains 5 red and black balls and another bag contains 2 red and 6 black balls. Two balls are drawn at random (without replacement) from one of the bags and both are found to be red. Find the probability that balls are drawn from first bag. [*CBSE 2019*]

Q.28 From the set {1, 2, 3, 4, 5}, two numbers a and $b(a \neq b)$ are chosen at random. The probability that $\frac{a}{b}$ is an integer is [*CBSE 2020*]

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{3}{5}$

Q.29 A bag contains 3 white, 4 block and 2 red balls. If balls are drawn at random (without replacement), then the probability that both the balls are white is

(A) $\frac{1}{18}$ (B) $\frac{1}{36}$ (C) $\frac{1}{12}$ (D) $\frac{1}{24}$

[*CBSE 2020*]

Q.30 An unbiased coin is tossed 4 times. Find the probability of getting atleast one head. [*CBSE 2020*]

Q.31 In a shop X-30 tins of ghee of type A and 40 tins of ghee of type B which look alike, are kept for sale. While in shop Y, similar 50 tins of ghee of type A and 60 tins of ghee of tyep B are there. One tin of ghee is purchased from one of the randomly selected shop and is found to be of type. Find the probability that it is purchased from shop Y. [*CBSE 2020*]

Q.32 A coin is biased so that the head is three times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails. Hence find the mean of the number of tails. [*CBSE 2020*]

Q.33 Find the probability distribution of the random variable X, which denotes the number of doublets in four throws of a pair of dice.
Hence, find the mean of the number of doublets (X). [*CBSE 2020*]